The Relaxation of Concentration Polarization in a Reverse Osmosis Desalination System

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An analysis of diffusional effects in a system consisting of impermeable relaxation sections placed alternately between semipermeable membrane sections indicates significant attenuation of salt concentration at the boundary in the impermeable section. The analysis employed exploits Equation (16), a relation deduced by Lighthill (4), to avoid the necessity of using orthogonal function expansions which converge slowly in the diffusion entrance region which is studied here.

The study of desalination of seawater has received considerable attention in recent years. Of the many possible processes considered as being potentially useful, the membrane process is currently regarded as one of the more promising, and a number of publications (*1* to *3*) have appeared recently in which brine side diffusion in continuous reverse osmosis systems was studied analytically. Although all of these analyses were carried out under various simplifying assumptions, and there have been some disagreements about the interpretation of the results of these investigations, all of these studies have demonstrated the detrimental effect of the salt buildup along the membrane boundary which is inherent in a reverse osmosis system. This salt buildup increases the effective osmotic pressure at the brine-membrane interface and thus reduces the pressure driving force and the productivity of the system.

In view of these findings, it is logical to consider ways by which the effect of salt buildup can be reduced. For example, this can be accomplished by a sudden change of the flow pattern of brine, thus producing a mixing effect by using artificial barriers inside the membrane conduit or by an abrupt reduction of the cross section. Because of the complexities, in both geometry and flow pattern, introduced by these devices, their effectiveness can be tested only through experimentation and no work in this regard has appeared in the literature up to the present time.

In a continuous reverse osmosis system, salt concentration buildup along the boundary is due to the movement of water across the membrane and the inability of salt to permeate. This results in the buildup of high salt concentrations (and concentration gradients) at the membrane boundary. A possible method of reducing the concentration polarization would be to place impermeable relaxation sections between membrane sections.

In an impermeable section, since there will be no movement of water in the transverse direction, the high concentration at the boundary is attenuated by molecular diffusion and convection which redistribute salt more uniformly across the flow. It is possible that by proper arrangement of the impermeable sections and membrane sections, one could obtain greater production capacity in a reverse osmosis system even though a fraction of the conduit is nonproductive. Furthermore, this process is simple enough to be analyzed theoretically; consequently, one can obtain reasonably accurate information on its effectiveness in reducing concentration polarization. The object of the present investigation is to study analytically the effectiveness of these impermeable sections for the reduction of concentration polarization. An analytical expression based on Lighthill's formula is given which predicts the relaxation of wall concentration in the impermeable sections. Also, approximate expressions are obtained for the wall concentration in the successive production and relaxation sections.

ANALYSIS

For simplicity it is assumed that the reverse osmosis system under investigation consists of a number of flat membrane sheets placed in parallel positions to form a flat duct with brine flowing inside. Impermeable sections are inserted between the membrane sections for the purpose of reducing the salt concentration polarization. A schematic drawing of this arrangement is shown in Figure 1. In Figure 1 only three sections are shown but this can be extended indefinitely with membrane and impermeable sections in alternating positions.

The local water production rate inside the membrane section (volume of water product per unit time per unit length of membrane with unit width) is assumed to be

$$\begin{aligned} -v_w &= A(\pi - P_{os}) \\ &= A\pi\left[1 - \frac{(P_{os})_o}{\pi}\frac{C_w}{C_o}\right] \end{aligned} \tag{1}$$

Furthermore, it is assumed that osmotic pressure is linearly proportional to the concentration.

It is clear that in order to provide answers to the problem posed in this investigation, it is desired to obtain the salt concentration distribution along the entire conduit, including both the membrane and impermeable sections.

The extent to which the salt concentration profile is relaxed provides a measure of the effectiveness of this arrangement. This information can be obtained from the solution of the diffusion equation written as

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{2}$$

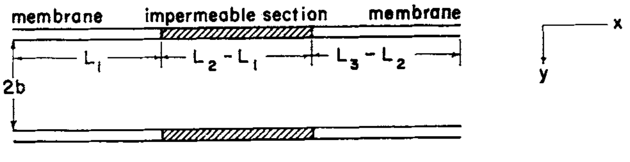

Fig. 1. Schematic diagram illustrating possible arrangement of membrane and impermeable relaxation sections. The distance *L* is measured from leading edge of first membrane section.

with the boundary conditions

$$C = C_o, \quad x \leqq 0, \quad \text{for all } y \tag{3}$$

$$\frac{\partial C}{\partial y} = 0, \quad y = b$$

and

$$-D\frac{\partial C}{\partial y} = (C)(A\pi)\left[1 - \frac{(P_{os})_o}{\pi}\frac{C}{C_o}\right] \text{ at } y = 0 \text{ for} \tag{3a}$$

$$0 < x < L_1 \text{ and } L_2 < x < L_3$$

and

$$\frac{\partial C}{\partial y} = 0, \; y = 0 \text{ for } L_1 < x < L_2 \tag{3b}$$

Equation (2) was written with the usual boundary-layer simplifications and the assumption that the physical properties are independent of salt concentration.

Equations (3*a*) and (3*b*) are written on the basis of no salt flux across the boundary of the conduit. Equation (3*a*) also implies that the membrane is perfect because it rejects the salt completely. Because of the difference in boundary conditions for the various sections, it is convenient to discuss the solutions separately.

Solution for the First Membrane Section

Previous investigations concerning the operation of reverse osmosis systems have been confined to problems of this type and a number of solutions are available. Perhaps the most accurate one is the series solution obtained by Gill, Tien, and Zeh (3) for the entrance region of the conduit. By assuming the flow of brine inside the conduit to be laminar and fully developed, the concentration distribution of salt was found to be

$$\frac{C}{C_o} = 1 + \sum_{k=1}^{\infty} \theta_k(\beta)\,\sigma^k \tag{4}$$

where

$$\sigma = \left[9\frac{(A\pi)^3\mu}{D^2}\int_o^x \frac{dx}{\tau_w(x)}\right]^{1/3}$$

$$\beta = \frac{A\pi y}{D\sigma}$$

With these definitions, for fully developed flow between parallel plates, Equation (2) reduces to the system

$$\theta''_k + 3\beta^2\theta'_k - 3k\beta\theta_k = (B_2 - 1)\theta_{k-1} + B_2\sum_{j=1}^{k-2}\theta_j(0)\theta'_{k-j-1} \tag{5}$$

and the boundary conditions are

$$\left.\begin{aligned} &\theta'_k(\infty) = 0 \text{ for all } k \\ &-\theta'_1(0) = 1 - B_2 \\ &-\theta'_k(0) = (1 - 2B_2)\,\theta_{k-1}(0) - B_2 \\ &\sum_{j=1}^{k-2}\theta_j(0)\theta_{k-j-1}(0) \text{ for } k \geqq 2 \end{aligned}\right\} \tag{6}$$

Numerical values of θ_k corresponding to various values of β for k up to 9 were tabulated and are given in references 3 and 6 and some values of the $\theta_k(0)$ are given in Table 1.

The shear stress at the wall τ_w for a fully developed laminar flow between parallel plates is well known to be

$$\tau_w = \frac{3\,U_b\mu}{b} \tag{7}$$

Accordingly, the variable σ is equivalent to

$$\sigma = \frac{(3)^{1/3}}{1 - B_2}\frac{v_w(0)}{U_b}\left[\frac{1}{\alpha^2}\frac{x}{b}\right]^{1/3} = \frac{(3)^{1/3}}{1 - B_2}z \tag{3}$$

where

$$z = \frac{v_w(0)}{U_b}\left[\frac{1}{\alpha^2}\frac{x}{b}\right]^{1/3} \tag{9}$$

$$B_2 = \frac{P_{os}(0)}{\pi}$$

$$v_w(0) = A\,\pi(1 - B_2) \tag{10}$$

$$\alpha = \frac{D}{U_b\,b} = \frac{1}{N_{Pe}} \tag{11}$$

The salt concentration at the membrane wall is given as

$$\frac{C_w}{C_o} = 1 + \sum_{k=1}^{\infty} B_k\,z^k \tag{12}$$

where

$$B_k = \theta_k(0)\left[\frac{(3)^{1/3}}{1 - B_2}\right]^k \tag{13}$$

It should be mentioned that a similar expression for the salt wall concentration was also obtained by using Lighthill's formula (3) with a somewhat different set of values of B_k, which was caused by the different assumptions involved in treating the transverse velocity component. It is known on the basis of a previous study (6) that this assumption causes one to predict a higher wall concentration than actually exists. However, if $B_2 \geqq 0.25$, the error is less than about 10%.

Solution for the First Impermeable Section

The problem presented here is essentially a Graetz problem with an arbitrary inlet concentration profile given by Equation (4) as

$$\frac{C(L_1, y)}{C_o} = 1 + \sum_{k=1}^{\infty}\theta_k\left(\frac{A\pi y}{D\sigma_{L_1}}\right)\sigma_{L_1}{}^k \tag{14}$$

where

$$\sigma_{L_1} = \frac{\sqrt[3]{3}}{1 - B_2}\frac{v_w(0)}{U_b}\left[\frac{1}{\alpha^2}\frac{L_1}{b}\right]^{1/3} \tag{15}$$

and with insulated boundary conditions

$$\frac{\partial C}{\partial y} = 0, \quad \text{at} \quad y = 0, \quad \text{and } b$$

The conventional approach to the solution of the Graetz problem involves the use of the separation of variables

TABLE 1. VALUES OF SERIES COEFFICIENTS $\theta_k(0)$

B_2	$\theta_1(0)$	$\theta_2(0)$	$\theta_3(0) \times 10$	$\theta_4(0) \times 10^2$	$\theta_5(0) \times 10^3$	$\theta_6(0) \times 10^4$	$\theta_7(0) \times 10^5$	$\theta_8(0) \times 10^6$	$\theta_9(0) \times 10^7$
0	0.7385	0.2407	0.4384	0.4446	0.1591	−0.1593	−0.1758	0.0466	0.1498
0.125	0.6462	0.1316	−0.0164	−0.3340	0.1309	5.435	6.251	−33.05	−80.42
0.25	0.5539	0.04509	−0.3261	−0.2926	3.075	2.033	−32.54	−13.29	363.7
0.5	0.3692	−0.06025	−0.1605	1.164	1.258	−13.24	59.47	29.22	−1061

technique and the subsequent matching of the initial condition with orthogonal expansion. It may appear at first that this approach should be employed in the present problem. However, further consideration reveals that a much more convenient procedure is feasible if one is interested primarily in the diffusion entrance region. The main disadvantage of the more traditional approach is that the initial condition [Equation (14)] in this case depends on the magnitude of L_1, and therefore it is necessary to evaluate the expansion coefficients for various values of L_1. Furthermore, the expansion coefficients of the concentration expression for a second relaxation section would depend on the length of all the previous sections, L_1, L_2 and L_3, and so on. The tediousness of the computation is compounded rapidly as more impermeable sections are added and this approach quickly becomes impractical. Furthermore, the series expansion used in the classical Graetz type of solution converges very slowly for small longitudinal distances and, therefore, is not convenient for describing the entrance region which is probably very important as rapid changes occur there in reverse osmosis systems.

In contrast, if one is interested in relaxing a profile which has developed over a large distance from the diffusion inlet, then the Graetz type of orthogonal expansion approach is most useful, since the technique to be described here becomes invalid in this region. However, except for the hypothetical case of $B_2 = 0$, no analytical solutions are presently available for large distances from the inlet of the membrane section.

Lighthill deduced the relation

$$\overline{\left(\frac{\partial \phi}{\partial y}\right)_w \left(\frac{2\,\mu}{\rho \tau_w}\right)^{1/2}} = -\,(2/3)^{2/3}\frac{(-1/3)!}{(1/3)!}\,p^{1/3}\,\bar{\phi}_w \tag{16}$$

where bars denote transformed functions of t, and p is the Laplace transform parameter with respect to the variable t. Clearly, if one notes that

$$p^{1/3}\,\bar{\phi}_w = \frac{p\,\bar{\phi}_w}{p^{2/3}}$$

by convolution Equation (16) is easily inverted to get

$$-\left(\frac{\partial C}{\partial y}\right)_w = \sqrt{\frac{\rho \tau_w}{2\mu}}\,\frac{(2/3)^{2/3}}{(1/3)!}\int_o^t \frac{\overline{\dfrac{dC_w}{dt_1}}}{(t-t_1)^{1/3}}\,dt_1 \tag{17}$$

where

$$t = \int_o^x \sqrt{\frac{2\mu\rho\tau_w}{{N_{Sc}}^2}}\,dx \tag{18}$$

Also, by simply transposing $p^{1/3}$, one can obtain

$$\phi_w = C_w - C_o$$

$$= \frac{(1/3)\,!\,(2/3)^{-2/3}}{\left(-\dfrac{1}{3}\right)!\,\Gamma\left(\dfrac{1}{3}\right)}\int_o^t \frac{-\left(\dfrac{\partial C}{\partial y}\right)_w \left(\dfrac{2\,\mu}{\rho\tau_w}\right)^{1/2}}{(t-t_1)^{2/3}}\,dt \tag{19}$$

Equations (17) and (19) are very useful in studying the relaxation of concentration polarization that occurs in the impermeable sections. Furthermore, these equations are very general and can obviously be used to study a much broader class of problems than the reverse osmosis systems considered here.

Equation (16) was obtained by linearizing the longitudinal velocity profile in the diffusion equation, which, for practical purposes, is valid in the diffusion entrance region for fluids with Schmidt numbers on the order of unity or greater, and this is the case for brine solutions. Another assumption implicit in Lighthill's Equation (16) is that the transverse velocity vanishes at the wall. For fully developed flow in a flat duct, this is exact in the relaxation section but introduces error in the membrane section.

The concentration gradient at the membrane surface can be obtained by combining Equations (7), (12), (17), and (18). To do this, first one can write

$$-\left(\frac{\partial C}{\partial y}\right)_w = \frac{1}{(1/3)!}\left(\frac{1}{3b^2\alpha}\right)^{1/3}\int_o^x \frac{(dC_w/dx_1)}{(x-x_1)^{1/3}}\,dx_1 \tag{20}$$

Then the integral can be reduced as follows by using Equation (12).

$$\int_o^x \frac{(dC_w/dx_1)}{(x-x_1)^{1/3}}\,dx_1$$

$$= \frac{v_w(0)}{U_b}\left[\frac{1}{b\alpha^2}\right]^{1/3}\int_o^z \frac{dC_w/dz_1}{[z^3-{z_1}^3]^{1/3}}\,dz_1$$

$$= \frac{v_w(0)}{U_b}\left[\frac{1}{b\alpha^2}\right]^{1/3} C_o \sum_{k=1}^{\infty} k\,B_k A_k\,z^{k-1} \tag{21}$$

where

$$A_k = \int_o^1 \frac{W^{k-1}}{(1-W^3)^{1/3}}\,dW = (1/3)\,\frac{\Gamma\left(\dfrac{k}{3}\right)\Gamma\left(\dfrac{2}{3}\right)}{\Gamma(k+2)} \tag{22}$$

Finally, by combining Equations (20) and (21), the surface concentration gradient is found to be

$$-\left(\frac{\partial C}{\partial y}\right)_w = \left(\frac{1}{3}\right)^{1/3}\frac{1}{(1/3)!}\,\frac{C_o\,v_w(0)}{b\,\alpha\,U_b}\sum_{k=1}^{\infty} k\,B_k\,A_k\,z^{k-1},\ 0 < z < z_{L_1} \tag{23}$$

where

$$z_{L_1} = \frac{v_w(0)}{U_b}\left[\frac{L_1}{\alpha^2 b}\right]^{1/3}$$

For the impermeable section, $L_1 < x < L_2$, one has

$$-\left(\frac{\partial C}{\partial y}\right)_w = 0,\quad z_{L_1} < z < z_{L_2} \tag{24}$$

and the surface concentration can be obtained by combining Equations (19), (23), and (24). First, Equation (19) can be written as

$$\gamma = \frac{C_w}{C_o} - 1 = \frac{b^{2/3}\alpha^{1/3}}{3^{2/3}\left(-\dfrac{1}{3}\right)!\,C_o}\int_o^x \frac{-\left(\dfrac{\partial C}{\partial y}\right)_w}{(x-x_1)^{2/3}}\,dx_1 \tag{25}$$

If one substitutes Equations (23) and (24) into Equation (25) and notes that the integral vanishes for $L_1 < x < L_2$, the surface concentration for the impermeable section $L_1 < x < L_2$ becomes

$$\gamma = \sum_{k=1}^{\infty}\frac{k\,A_k\,B_k\,B_\epsilon\left(\dfrac{k+2}{3},\dfrac{5}{3}\right)}{\left(-\dfrac{1}{3}\right)!\,\left(\dfrac{1}{3}\right)!}\,z^k \tag{26}$$

where

$$\epsilon = \left(\frac{L_1}{x}\right),\ \text{and}\ B_\epsilon\left(\frac{k+2}{3},\frac{5}{3}\right)$$

is an incomplete beta function defined by

$$B_\epsilon\,(p, q) = \int_0^\epsilon t^{p-1}\,(1-t)^{1-q}\,dt, \quad 0 \leqq \epsilon \leqq 1 \qquad (27)$$

and is tabulated in reference 7.

Solution for the Second Membrane Section and Subsequent Sections

The procedure discussed above, in principle, can be applied to determine the surface concentration distribution in the second membrane section, but the relevant integral equation is very complicated. Thus, an alternate approximate procedure for this and subsequent sections will be discussed which is based on the use of appropriate limiting forms of the entrance concentration distributions to the respective membrane sections.

Clearly, in any cross section the maximum concentration exists at the wall and the minimum at the center of the channel. Furthermore, the minimum concentration is always greater than C_o, since pure water has been produced by the system. Thus, the point concentration anywhere in the stream entering the second membrane section at $x = L_2$ will be between the two uniform concentration distributions $C = C_o$ and $C = C_w(L_2)$. Consequently the true wall concentration distribution in the membrane section will be bounded by those longitudinal distributions obtained by using these two limiting uniform radial distributions as entrance conditions. Furthermore, a conservative result, in the sense that the predicted wall concentration will be equal to or greater than the true one, is obtained by using $C(L_2, y) = C_w(L_2)$ at the entrance to the second membrane section. Thus

$$\frac{C_w}{C_w(L_2)} = 1 + \sum_{k=1}^{\infty} B_k\,(z^3 - z^3{}_{L_2})^{\frac{k}{3}}$$

or

$$\frac{C_w}{C_o} = \frac{C_w(L_2)}{C_o}\left[1 + \sum_{k=1}^{\infty} B_k\,(z^3 - z^3{}_{L_2})^{k/3}\right]$$

$$= \left\{1 + \sum_{k=1}^{\infty} \frac{k\,A_k\,B_k\,B_{\epsilon_1}\left(\frac{k+2}{3}, \frac{5}{3}\right)}{\left(\frac{1}{3}\right)!\,\left(\frac{1}{3}\right)!}\,{z_{L_2}}^k\right\}$$

$$\left[1 + \sum_{k=1}^{\infty} B_k\,(z^3 - z^3{}_{L_2})^{\frac{k}{3}}\right] \qquad (28)$$

with $\epsilon_1 = L_1/L_2$.

Also the approximate distribution for the second relaxation section can be written down immediately by inspection by using Equation (26) if one recognizes that

$$C_o \rightarrow C_w(L_2),\ L_1 \rightarrow L_3 - L_2,\ x \rightarrow x - L_2,\ z \rightarrow (z^3 - z^3{}_{L_2})^{1/3}$$

and the result is

$$\frac{C_w}{C_o} = \left\{1 + \sum_{k=1}^{\infty} \frac{k\,A_k\,B_k\,B_{\epsilon_1}\left(\frac{k+2}{3}, \frac{5}{3}\right)}{3\left(\frac{1}{3}\right)!\,\Gamma\left(\frac{1}{3}\right)}\,{z_{L_2}}^k\right\}$$

$$\left\{1 + \sum_{k=1}^{\infty} \frac{k\,A_k\,B_k\,B_{\epsilon_2}\left(\frac{k+2}{3}, \frac{5}{3}\right)}{\left(-\frac{1}{3}\right)!\,\left(\frac{1}{3}\right)!}\,(z^3 - z^3{}_{L_2})^{\frac{k}{3}}\right\} \qquad (29)$$

where

$$\epsilon_2 = \frac{L_3 - L_2}{x - L_2}$$

Obviously, this procedure can be carried out indefinitely to obtain approximate solutions for the successive sections. However, the results get progressively more complicated and inaccurate. Thus it doesn't seem as though it would be really fruitful to carry this out further than, say, the third relaxation section.

DISCUSSION OF RESULTS

The extent to which the concentration polarization can be relaxed is shown in Figure 2 for the case of $B_2 = ½$. The series solution expression obtained earlier (3) was used for the expression of salt buildup in membrane section and Equation (26) was used for the surface concentration change in the impermeable sections for various values of the length of the membrane section σ_{L_1}. It should be kept in mind that the physical length x is proportional to the cube power of σ and the relative distance for $\sigma = 1$ to $\sigma = 2$ is seven times that for $\sigma = 0$ to $\sigma = 1$.

As an indication of the effectiveness of these impermeable sections, a relaxation efficiency is defined as the percentage reduction of the excess salt concentration or

$$E = \frac{C_{w1} - C_{w2}}{C_{w1} - C_o} \qquad (30)$$

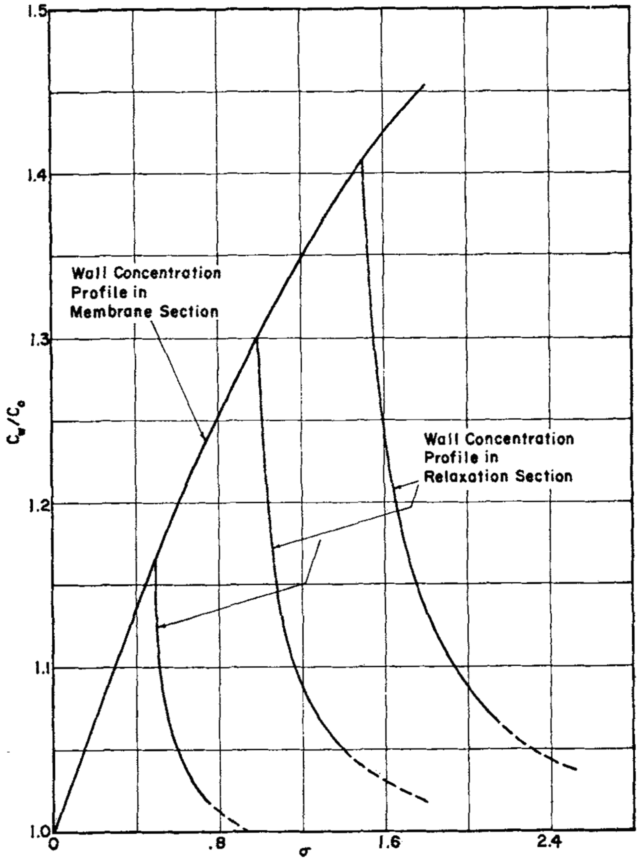

Fig. 2. Wall concentration distribution in first membrane and impermeable relaxation sections as a function of σ for $B_2 = 0.5$. The dimensionless variable is given by $\sigma = \frac{v_w(0)}{U_b(1-B_2)}\left[\frac{3}{\alpha^2}\frac{x}{b}\right]^{1/3}$ with $\alpha = (D/bU_b)$.

where C_{w1} is the value obtained from Equation (12) at $x = L_1$, and C_{w2} is the exit concentration for the impermeable section which can be obtained from Equation (26). This quantity depends on two parameters: the length of the membrane section (σ_{L1} or z_{L1}) and the relative length of the impermeable section to the preceding membrane section.

or

$$\frac{L_2 - L_1}{L_1} = \frac{{z_{L2}}^3 - {z_{L1}}^3}{{z_{L1}}^3} = \frac{{\sigma_{L2}}^3 - {\sigma_{L1}}^3}{{\sigma_{L1}}^3}$$

The results are shown graphically in Figure 3.

It is of interest to compare the rate of pure water production obtained from systems with intervening impermeable sections, as shown in Figure 1, with those consisting of continuous semipermeable membranes along the total conduit length. One obtains

$$Q_2 = \int_o^{L_3} A\pi \left[1 - B_2 \frac{C_w}{C_o} \right] dx, \tag{31}$$

and

$$Q_1 = \int_o^{L_1} A\pi \left[1 - B_2 \frac{C_w}{C_o} \right] dx + \int_{L_2}^{L_3} A\pi \left[1 - B_2 \frac{C_w}{C_o} \right] dx \tag{32}$$

The expression of (C_w/C_o) given by Equation (12) will be used in Equation (31) and in the first integral of Equation (32). However, Equation (26) applies to the second membrane section $L_3 - L_2$; therefore it is used in the second integral. A number of calculations were made for various combinations of L_1, L_2, and L_3 and the rates of production of pure water computed from Equations (31) and (32) were compared. In all these cases Q_1 was found to be less than Q_2 within the applicable range of Equation (12), namely, in the diffusional entrance region of the conduit. This would not necessarily be the case for long membrane sections, but such systems cannot be studied conveniently by the present method of analysis.

These findings were not unexpected, since examination of the concentration polarization relaxation pattern in Figure 2 shows, for all the cases studied, that the length required for a given reduction in wall concentration is always higher than the corresponding length of the membrane section for the same increase in concentration buildup. This point can also be illustrated in a different way. For example, if the impermeable section is taken to be of the same length as the membrane section, $(L_2 - L_1)/(L_1) = 1$, Figure 3 shows that the percent reduction of the excess concentration at the exit of the impermeable section is only 64% for $\sigma_{L1} = 2$ and 78% for $\sigma_L = 0.5$.

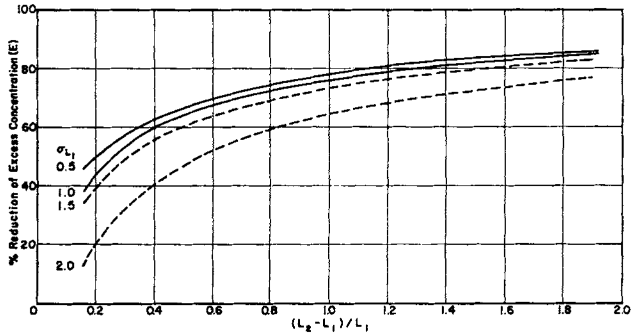

Fig. 3. Percentage reduction of excess salt concentration for $B_2 = 0.5$ with dimensionless length of first membrane section as parameter.

It appears that the reduction of concentration polarization caused by impermeable sections, which would normally be used in constructing semipermeable conduits to strengthen and support membrane sections, can be significant. The attractiveness of such reduction is enhanced in practical systems which involve imperfect membranes that do not completely reject salt. As pointed out by Sherwood et al. (*1*) polarization is even more deleterious when salt rejection is not complete, since it increases the salinity of the product water. Another condition which may be of major importance is the extent of the effect of high salinity on membrane life.

The method employed here to study reverse osmosis effects can, of course, be employed to study other phenomena such as the determination of surface concentrations in systems with catalytic walls and finite interfacial velocity. A variety of heat transfer and heterogeneous reaction problems can also be approached from this point of view. Whether one uses series expansions similar to Equations (4), (5), and (6), and/or the results based on Lighthill's relation depends on the problems under consideration.

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NOTATION

A_k = constants defined by Equation (22)
A = membrane constant
b = half width of the conduit
B_2 = constant defined by Equation (8)
B_k = expansion coefficients in Equation (12) and is defined by Equation (13)
$B_\epsilon(p, q)$ = incomplete beta function
C = concentration of salt
C_o = initial salt concentration
C_w = salt concentration at the boundary of the conduit
C_k = quantities defined by Equation (27)
D = diffusion coefficient of salt in brine solution
E = efficiency of reducing excess salt concentration defined by Equation (30)
L_1, L_2, L_3 = length measured from the beginning to the exits of the various membrane and impermeable sections
P_{os} = osmotic pressure of brine
$(P_{os})_o$ = osmotic pressure of brine at concentration C_o
Q_1, Q_2 = rate of production of pure water defined by Equations (31) and (32)
N_{Sc} = Schmidt number
t = variable defined by Equation (18)
U_b = average bulk velocity of brine inside the conduit
u = velocity component along x direction
v = velocity component along y direction
v_w = wall velocity
$v_w(o)$ = wall velocity at $x = 0$
x = longitudinal distance
y = distance measured away from the boundary

Greek Letters

α defined as $(D/U_b\, b)$
β defined as $(A\pi y/D\sigma)$
γ = excess salt concentration defined by Equation (25)
$\Gamma(x)$ = gamma function
ρ = density
σ defined by Equation (5)
π = total pressure
θ_k = functions used to describe C_w, Equation (4)
τ_w = shear stress at wall
μ = viscosity of brine

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Film Boiling of Liquid Nitrogen from Porous Surfaces with Vapor Suction: Experimental Extensions

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Previous work on film boiling of liquid nitrogen on flat horizontal porous surfaces with porous ceramic flow control elements is extended to tubular geometries and to very thin (asbestos paper) flow control members. At no loss in stability it is demonstrated that heat transfer coefficients more than five times greater than those in conventional film boiling can be obtained. An approximate model for a single liquid tongue breaking through the heating section at the critical flow rate is also presented. The upper (complete breakthrough) and lower (local breakthrough) limits on critical flow rate can be obtained theoretically. The experimental points lie between these two limits and, in general, are fairly close to the upper limit.

Methods for improving the heat transfer rate in both the stable boiling regimes (nucleate and film boiling) have been the subject of a number of investigations in recent years. These include the use of capillary wicking (*1* to *3*), fixed nucleation sites (*4*, *5*), impressed electrical fields (*6* to *9*), ultrasonic fields and pulsating pressures (*10* to *12*), and porous heating sources with vapor suction (*13* to *16*). The last mentioned mode of boiling heat transfer, with which the present work is concerned, provides the additional advantage of vapor superheat and the elimination of vapor binding in reduced gravitational fields.

The feasibility of film boiling* on an electrically heated horizontal porous plate with the vapor being sucked through the plate has been demonstrated in previous studies (*14*, *15*). In order to stabilize the system, it was necessary to use a porous glass-bonded quartz block, 3/16 to 1½ in. thick, next to the heating element on the liquid side. In the absence of this element, strong pressure oscillations were recorded, followed by quenching of a portion of the heating element to the liquid temperature. Increases in heat transfer coefficients of about 2.5 times over those in normal film boiling were noted with vapor suction. Although the ceramic blocks were found to be well able to stabilize film boiling with vapor suction, the full advantages of this mode of heat transfer were not achieved. With no vapor suction, all the vapor was vented up through the ceramic blocks, resulting in considerably lower heat transfer coefficients. Change in porosity of the blocks was shown to have negligible effects on the heat transfer coefficient, although there existed a limiting porosity for effective flow control. It seemed therefore desirable to investigate other flow control elements over a wide range of heat fluxes. In the present work, film boiling of liquid nitrogen with vapor suction is studied on both flat and tubular porous heating elements at heat fluxes as high as 80,000 B.t.u./(hr.)(sq.ft.). A porous metallic plate, which is similar to the heating element and hence provides a compact sandwich construction, as well as thin asbestos papers, are used as flow control elements. Two different types of materials varying in thickness are used for the heating elements. In addition, an approximate theoretical model for liquid breakthrough is developed in the Appendix.

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* Actually, none of the common boiling regime classifications fit the present system. The term *controlled-access film boiling* might be more properly descriptive, since at high suction rates it is thought that the flow control elements serve to distribute liquid uniformly to the hot porous surface. At sufficiently low flow rates, however, vaporization undoubtedly occurs principally in the flow control element.

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